SRIP: A SAT-based System for Independent Set Reconfiguration

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KR in the Wild — Session: SAT Encoding

What is Combinatorial Reconfiguration (CR)?

Input

1. Combinatorial problem P
2. Reconfiguration rule
3. X_s: solution of P
4. X_g: solution of P

Output

A sequence of solutions of P from X_s to X_g while satisfying a given reconfiguration rule.

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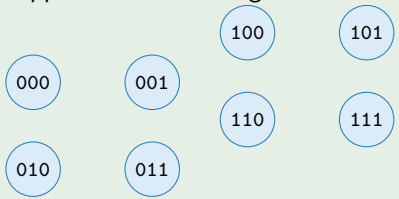
Example

Suppose that P has eight solutions.



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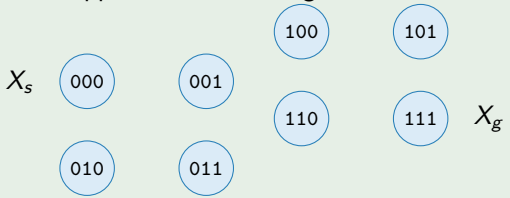

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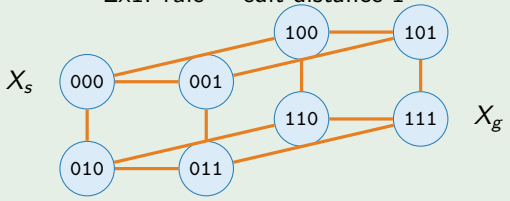
Ex1. rule = edit distance 1



Valid sequence: 000 → 001 → 101 → 111

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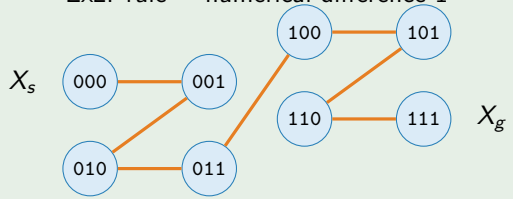
Example

Ex2. rule = numerical difference 1



Valid sequence: 000 → 001 → 010 → 011 → 100 → 101 → 110 → 111

Many Reconfiguration Problems Are PSPACE-Complete

Many NP-complete problems have PSPACE-complete reconfiguration counterparts.

- SAT Reconfiguration [Gopalan+, 2009, Mouawad+, 2017]
- Coloring Reconfiguration [Bonsma+, 2009, Brewster+, 2016, Cereceda+, 2011]
- Dominating Set Reconfiguration [Haddadan+, 2016, Suzuki+, 2016]
- Clique Reconfiguration [Ito+, 2015]
- Hamiltonian Cycle Reconfiguration [Takaoka, 2018]
- Set Covering Reconfiguration [Ito+, 2011]
- Independent Set Reconfiguration [Ito+, 2011, Kaminski+, 2012]

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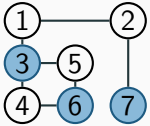
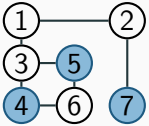
Among them, the Independent Set Reconfiguration Problem (ISRP) is one of the most extensively studied problems in the field.

Independent Set Reconfiguration under Token Jumping Rule (ISRP)



I_s



I_g

Input of ISRP

- Independent Set Problem: $G = (V, E)$
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  (exact one token can jump)
- I_s: independent set
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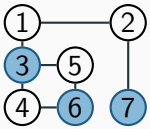
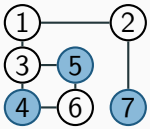


$I_s = I_0$



$I_3 = I_g$

Input of ISRP

- Independent Set Problem: $G = (V, E)$
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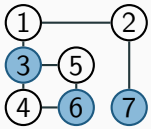
- I_s: independent set
- I_g: independent set

Query.

Is there a solution sequence from I_s to I_g?

Independent Set Reconfiguration under Token Jumping Rule (ISRP)



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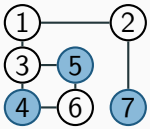

$I_3 = I_g$

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  (exact one token can jump)
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Output

$I_s = I_0$ $\{3, 6, 7\}$ 0010011

$I_g = I_3$ $\{4, 5, 7\}$ 0001101

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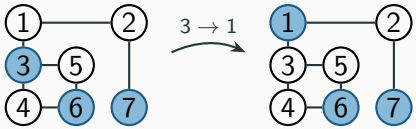
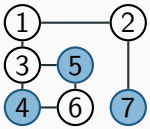


$I_s = I_0$ I_1



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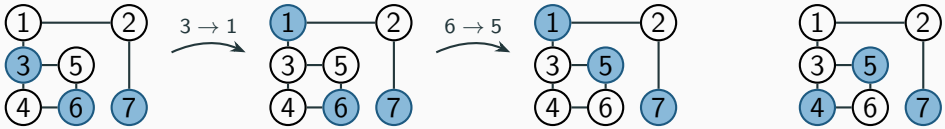

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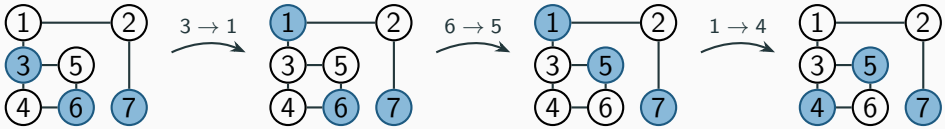

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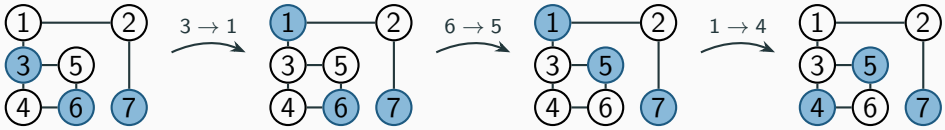

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ISRP is PSPACE-complete.

CoRe Challenges Established ISRP Benchmarks and Solvers

- Until around 2020, studies on ISRP focused mainly on theoretical complexity.
- The international CoRe Challenges were held in 2022 and 2023, establishing shared benchmark instances and state-of-the-art practical solvers.
- The competitions compared solvers based on AI planning [Christen+, 2023], SAT-based BMC [Froleyks+, 2022], ASP-based BMC (recongo) [Yamada+, 2024], decision diagrams [Ito+, 2023], and BFS-based search.

In 2023, recongo produced a shortest reconfiguration sequence for 433 instances and ranked first in the single-engine #Shortest category.

Goal: Find Shortest ISRP Sequences at Scale

Goal

Develop a high-performance ISRP solver for finding shortest reconfiguration sequences.

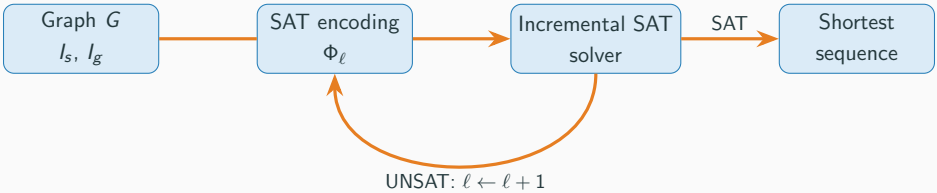
Methodology

- Use bounded model checking (BMC) with incremental SAT solving.
- Incremental SAT alone is insufficient: the encoding suffers from clause explosion as the bound and graph size increase.

Key ideas

1. Exploit the graph's clique structure to reduce the number of clauses.
2. Add optional sound constraints that prune redundant moves at the lower bound.

SRIP Uses Incremental SAT-Based BMC



1. Encode the input (G, I_s, I_g) as Φ_ℓ and pass it to the incremental SAT solver.
2. If UNSAT, increment ℓ, extend the encoding, and repeat while reusing clauses and learned information.
3. If SAT, extract a sequence at the current bound; the first SAT answer is shortest.

SAT encoding for a sequence of length ℓ

Let x_v^t be true iff vertex v has a token at time step t.

$$\Phi_\ell \;=\; \mathit{Init}(X^0) \wedge \mathit{Goal}(X^\ell) \wedge \bigwedge_{t=1}^{\ell-1} \mathit{Token}(X^t) \wedge \bigwedge_{t=0}^{\ell-1} \mathit{Jump}(X^t, X^{t+1}).$$

$$\mathit{Init}(X^0) \;=\; \bigwedge_{v \in I_s} x_v^0 \wedge \bigwedge_{v \notin I_s} \neg x_v^0,$$

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$$\mathit{Token}(X^t) \;=\; \bigwedge_{\{u,v\} \in E} (\neg x_u^t \vee \neg x_v^t) \wedge \sum_{v \in V} x_v^t = k$$

$$\mathit{Jump}(X^t, X^{t+1}) \;=\; \bigwedge_{v \in V} \left(p_v^t \leftrightarrow (x_v^t \wedge \neg x_v^{t+1})\right) \wedge \sum_{v \in V} p_v^t = 1.$$

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$$\begin{aligned} \mathit{Init}(X^0) &= \bigwedge_{v \in I_s} x_v^0 \wedge \bigwedge_{v \notin I_s} \neg x_v^0, \\ \mathit{Goal}(X^\ell) &= \bigwedge_{v \in I_g} x_v^\ell \wedge \bigwedge_{v \notin I_g} \neg x_v^\ell. \end{aligned}$$

It ensures the initial and goal states.

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It ensures that $|I_s| = |I_g| = k$ tokens consist an independent set in every state.

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It enforces the token-jumping rule.

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This part requires $O(\ell|V|k)$ clauses.

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The Exact-k Constraint Dominates Clause Growth

Let n be the number of vertices $|V|$, and m be the number of elements of clique partition of G.

Component	Per step	Total
Basic formula Φ_ℓ		
Init + *Goal*	—	$O(n)$
Independence	$O(\|E\|)$	$O(\ell\|E\|)$
Exact-k (n vars)	$O(nk)$	$O(\ell nk)$
Jump (link + exact-1)	$O(n)$	$O(\ell n)$
Total		$O\big(\ell(\|E\| + nk)\big)$

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Clique-based formula Φ_ℓ^{CC} **Idea 1**		
Clique linking	$O(n)$	$O(\ell n)$
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replace n by m

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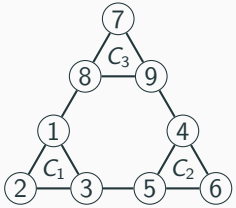
Idea 1: Count Occupied Cliques Instead of Vertices



- $G = (V, E)$ is given, where $V = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
- Suppose that a clique partition $\mathcal{P} = \{C_1, C_2, C_3\}$ is obtained, where $C_1 = \{1, 2, 3\}$, $C_2 = \{4, 5, 6\}$, and $C_3 = \{7, 8, 9\}$.

Basic formula Φ_ℓ

$$x_1^t + x_2^t + x_3^t + \cdots + x_7^t + x_8^t + x_9^t = k.$$

Clique-based formula Φ_ℓ^{CC}

$$c_1^t \leftrightarrow (x_1^t \vee x_2^t \vee x_3^t),$$
$$c_2^t \leftrightarrow (x_4^t \vee x_5^t \vee x_6^t),$$
$$c_3^t \leftrightarrow (x_7^t \vee x_8^t \vee x_9^t),$$
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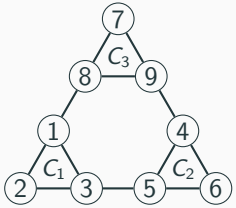

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Basic formula Φ_ℓ

$$x_1^t + x_2^t + x_3^t + \cdots + x_7^t + x_8^t + x_9^t = k.$$

We do not need to count tokens over all vertices; count occupied cliques instead!

Clique-based formula Φ_ℓ^{CC}

$$c_1^t \leftrightarrow (x_1^t \vee x_2^t \vee x_3^t),$$
$$c_2^t \leftrightarrow (x_4^t \vee x_5^t \vee x_6^t),$$
$$c_3^t \leftrightarrow (x_7^t \vee x_8^t \vee x_9^t),$$
$$c_1^t + c_2^t + c_3^t = k.$$

A Fast Heuristic Builds Small, Balanced Clique Partitions

Algorithm 1: `BestFitPartition`

Input: Graph $G = (V, E)$
Output: Clique partition $\mathcal{P} = \{C_1, \ldots, C_m\}$ of V

$\tau \leftarrow$ `EstimateMaxCliqueSize`(G);
$\mathcal{P} \leftarrow \emptyset$;
foreach $v \in V$ *in decreasing degree order* **do**
  best_index $\leftarrow \perp$;
  best_score $\leftarrow -\infty$;
  // find best-fit clique for v
  foreach $C_i \in \{C \in \mathcal{P} \mid \forall u \in C,\ \{u, v\} \in E\}$ **do**
    score $\leftarrow \tau - |C_i|$;
    if *score* $>$ *best_score* **then**
      best_index $\leftarrow i$;
      best_score $\leftarrow$ *score*;
  if *best_index* $\neq \perp$ **then**
    $C_{\text{best\_index}} \leftarrow C_{\text{best\_index}} \cup \{v\}$;
  else
    $\mathcal{P} \leftarrow \mathcal{P} \cup \{\{v\}\}$;
return $\mathcal{P}$;

Algorithm 2: `Rebalancing`

Input: Clique partition $\mathcal{P} = \{C_1, \ldots, C_m\}$ and graph $G = (V, E)$
Output: Rebalanced clique partition $\mathcal{P}$

repeat
  changed $\leftarrow$ **false**;
  foreach $C_s \in \mathcal{P}$ *in increasing order of* $|C_s|$ **do**
    foreach $v \in C_s$ **do**
      $\mathcal{F} \leftarrow \{C \in \mathcal{P} \mid \forall u \in C,\ \{u, v\} \in E \wedge |C| \geq |C_s|\}$;
      if $\mathcal{F} \neq \emptyset$ **then**
        $C_s \leftarrow C_s \setminus \{v\}$;
        $C_t \leftarrow \arg\min_{C \in \mathcal{F}} |C|$;
        $C_t \leftarrow C_t \cup \{v\}$;
        changed $\leftarrow$ **true**;
until *changed* $=$ **false**;
$\mathcal{P} \leftarrow \{C \in \mathcal{P} \mid C \neq \emptyset\}$;
return $\mathcal{P}$;

Together, these algorithms produce clique partitions with relatively few cliques and well-balanced clique sizes.

A Fast Heuristic Builds Small, Balanced Clique Partitions

Algorithm 3: `BestFitPartition`

Input: Graph $G = (V, E)$
Output: Clique partition $\mathcal{P} = \{C_1, \ldots, C_m\}$ of V

$\tau \leftarrow$ `EstimateMaxCliqueSize`(G);
$\mathcal{P} \leftarrow \emptyset$;
foreach $v \in V$ *in decreasing degree order* **do**
  best_index $\leftarrow \perp$;
  best_score $\leftarrow -\infty$;
  // find be… -fit clique for v
  foreach C_i … $\{C \in \mathcal{P} \mid \forall u \in C_i : \{u, v\} \in$ …
    score … $\tau - |C_i|$;
    if *sco…* > *best_score* **then**
      best_index $\leftarrow i$;
      best_score $\leftarrow$ *score*;
  if *best_index* $\neq \perp$ **then**
    $C_{best\_index} \leftarrow C_{best\_index} \cup \{v\}$;
  else
    $\mathcal{P} \leftarrow \mathcal{P} \cup \{\{v\}\}$;
return $\mathcal{P}$;

Algorithm 4: `Rebalancing`

Input: Clique partition $\mathcal{P} = \{C_1, \ldots, C_m\}$ and graph $G = (V, E)$
Output: Rebalanced clique partition $\mathcal{P}$

repeat
  changed $\leftarrow$ **false**;
  foreach $C_s \in \mathcal{P}$ *in increasing order of* $|C_s|$ **do**
    foreach $v \in C_s$ **do**
      $\mathcal{P} \leftarrow \{C \in \mathcal{P} \mid \forall u \in C : \{u, v\} \in$ …
      $|C| \geq |C_s|\}$;
      if … **then**
        $C_s \leftarrow C_s \setminus \{v\}$;
        $C_t \leftarrow \arg\min_{C \in \mathcal{P}} |C|$;
        $C_t \leftarrow C_t \cup \{v\}$;
        changed $\leftarrow$ **true**;
until *changed* $=$ **false**;
$\mathcal{P} \leftarrow \{C \in \mathcal{P} \mid C \neq \emptyset\}$;
return $\mathcal{P}$;

If you are interested, please see the paper for details.

Together, these algorithms produce clique partitions with relatively few cliques and well-balanced clique sizes.

Idea 2: Prune Redundant Moves at the Lower Bound

- Let $d = |I_s \setminus I_g|$. At BMC step $\ell = d$, no redundant move is possible.
- That is, tokens are just moved from $I_s \setminus I_g$ to $I_g \setminus I_s$ with an appropriate order.

Token fixation

Tokens on $I_s \cap I_g$ remain fixed at every step. No vertex outside $I_s \cup I_g$ can hold a token.

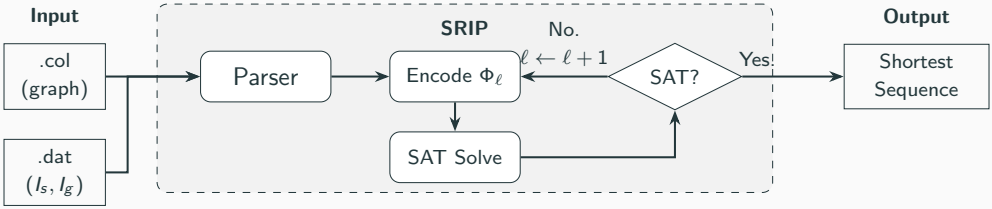
$$\bigwedge_{v \in I_s \cap I_g} \bigwedge_{t=0}^{d} x_v^t \wedge \bigwedge_{v \notin I_s \cup I_g} \bigwedge_{t=0}^{d} \neg x_v^t.$$

Monotonicity

A token that arrives at a goal-only vertex cannot leave; a token leaving a start-only vertex cannot return.

$$\bigwedge_{v \in I_g \setminus I_s} \bigwedge_{t=0}^{d-1} \left(x_v^t \rightarrow x_v^{t+1}\right) \wedge \bigwedge_{v \in I_s \setminus I_g} \bigwedge_{t=0}^{d-1} \left(\neg x_v^t \rightarrow \neg x_v^{t+1}\right).$$

SRIP Implementation: Incremental SAT with CaDiCaL



- Rust implementation with CaDiCaL 3.0 as the SAT backend
- We use sequential counters [Sinz, 2005] to encode exact-1 constraints.
- We use totalizers [Bailleux+, 2003] to encode clique-level exact-k.
- Extract and validate a reconfiguration sequence from the SAT model

Experiments and Setup

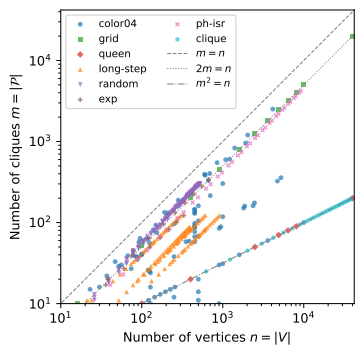
Experiments

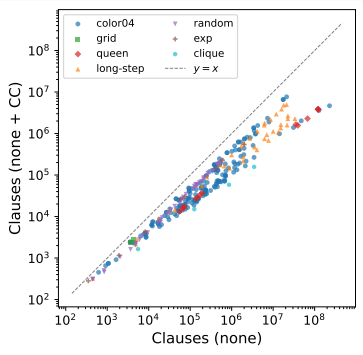
- Exp1. Evaluation of the reduction of the clique-based formula
- Exp3. Comparison with SOTA on CoRe Challenge 693 instances.
  - BMC solvers: recongo [Yamada+, 2024], ReconfAIGERation [Froleyks+, 2022]
  - AI planner: PARIS [Christen+, 2023]
- *Supplemental: Exp2. Ablation on 40 handcrafted clique-rich instances*

Set up

- Environment: Intel Xeon Platinum 8480+ (2.00 GHz), 128 GB RAM, Ubuntu 20.04; one core per instance
- 1800-second timeout
- #best: number of instances where a solver finds the minimum returned length
  - SRIP guarantees optimality; some SOTA solvers return best-so-far solutions.

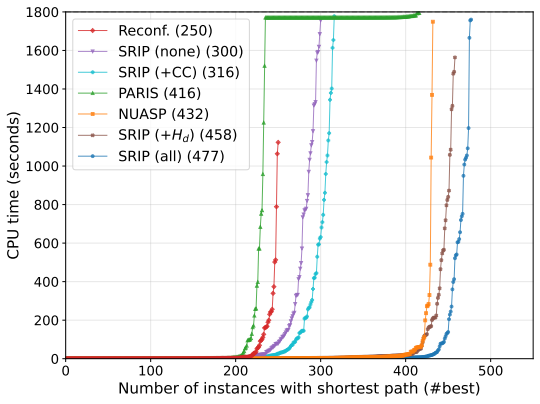
Exp1. Clique partitioning consistently reduces encoding size





- Median compression: $m/n \approx 0.45$; on the largest instances, $m/n = 0.005$ (a 200× reduction).
- At each shortest solved bound, clauses decrease on all 296 comparable instances by about 3–10× at the median.

Exp3. SRIP Finds Shortest Sequences on 477/693 Instances



Result

SRIP with $+H_d + CC$ finds shortest reconfiguration sequences for 477 of 693 instances, the highest count among the compared solvers.

Conclusion and future work

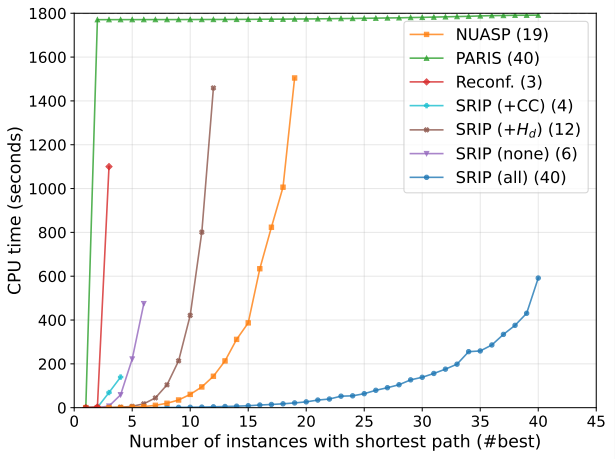
1. SRIP solves ISRP under Token Jumping with **SAT-based BMC** and returns shortest reconfiguration sequences.
2. **Clique-partition-based cardinality encoding** replaces vertex-level exact-k constraints with clique-level constraints, substantially reducing clauses.
3. **Token fixation and monotonicity** constraints strongly prune the search at the lower bound.
4. SRIP finds shortest sequences for **477 of 693** CoRe Challenge instances, the highest number in our comparison.

Future work

Better clique partitions, sound pruning beyond $\ell = d$, and applications to other reconfiguration problems.

Supplemental Slides

Exp2. Ablation: the two techniques are complementary



#best on 40 `clique` instances

Configuration	#best
$+H_d + CC$	40
$+H_d$	12
$+CC$	4
baseline	6

With both clique-based encoding (CC) and pruning constraints (H_d), SRIP obtains shortest sequences on every instance.

Backup: How much does partition quality matter?

BestFit + Rebalancing vs. naive greedy

	Proposed	Naive
Median number of cliques	82	95
Mean size std. deviation	1.53	2.78
Instances solved (of 693)	477	465

Cost

Preprocessing was negligible for both methods: about 0.002 seconds. The performance difference therefore appears in SAT encoding and solving.

Why not minimize the clique count exactly?

On 101 `color04` graphs, OR-Tools CP-SAT:

- proved optimality on only 21,
- returned unverified partitions on 68,
- failed within the limits on 12.

Answer

Partition quality matters, including balance beyond clique count, but exact minimization is too expensive as preprocessing.

Backup: What happens on unreachable instances?

Current behavior

An UNSAT result at bound ℓ rules out a sequence up to the bounds tested. SRIP then increases ℓ; without a practical completeness bound, an unreachable instance normally reaches the global timeout.

Role of H_d

Token fixation and monotonicity are active only at $\ell = d$. They strengthen the first UNSAT test, but they do not constitute a general proof of unreachability.

Future direction

Combine BMC with diameter or invariant bounds, IC3/PDR, or symbolic reachability. ReconfAIGERation and ZDD-based solvers already provide useful reference approaches for proving unsolvability.

Backup: Can the ideas generalize?

- Incremental BMC with block literals is problem-independent once states, goals, and transitions have SAT encodings.
- Clique-based cardinality applies when a state is a fixed-size subset with pairwise conflicts; examples include set packing and matching via a line graph.
- For Token Sliding, the state and clique encodings remain unchanged; the transition additionally requires the source and destination to be adjacent.
- Lower-bound pruning can transfer when each move makes at most one unit of progress toward the goal, but its constraints must be derived for each reconfiguration rule.

Limitation

Problems such as coloring have different state structure and require a different compression argument; the current clique encoding is not plug-and-play.

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